Compact Routing

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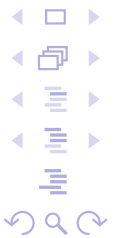
GI-Dagstuhl-Forschungsseminar Nr. 05473,
Algorithmen für Sensor- und Ad-hoc-Netzwerke,
23.-25. November 2005,
IBFI Schloss Dagstuhl

Outline



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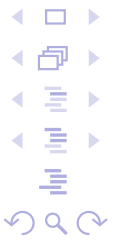
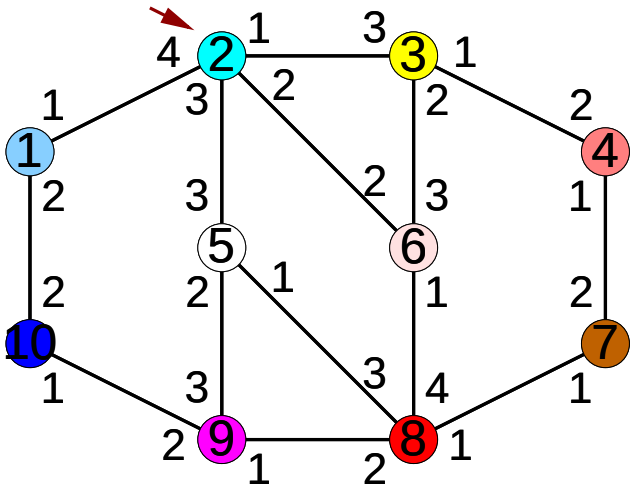
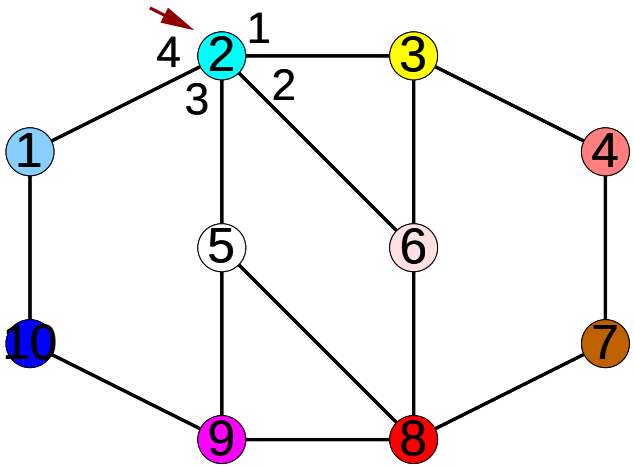


Routing: First Examples



7
DATA

Routing: First Examples

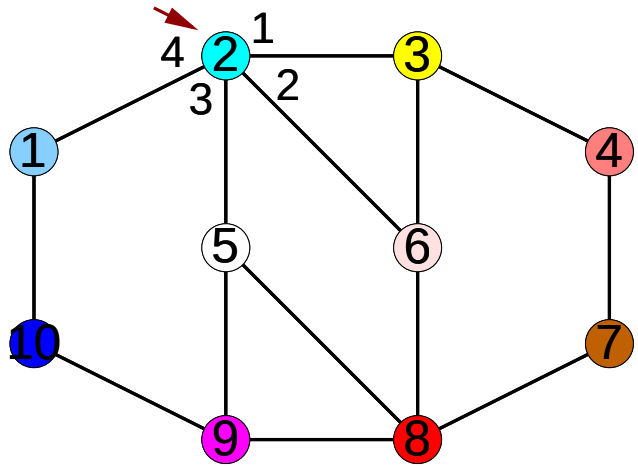


7
DATA

Local memory of node 2**:**	
Own address:	2
Port for node 1:	4
Port for node 3:	1
Port for node 4:	1
Port for node 5:	3
Port for node 6:	2
Port for node 7:	1
Port for node 8:	2
Port for node 9:	3
Port for node 10:	4

- Local memory:
  $n \cdot \log(\deg(v))$ bits

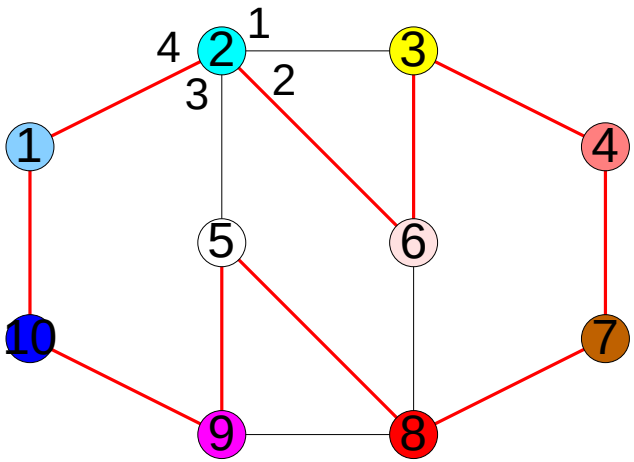
Routing: First Examples



Local memory of node 2:	
Own address:	2
Port 1:	3–4
Port 2:	6–8
Port 3:	5
Port 4:	9–1

- Local memory: $\deg(v) \cdot 2 \log n$ bits

Routing: First Examples



Local memory of node 2**:**	
Own address:	2
Port 2 for every destination.	

- Local memory:
  $\log n + \log(\deg(v))$ bits

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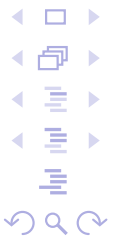
DATA

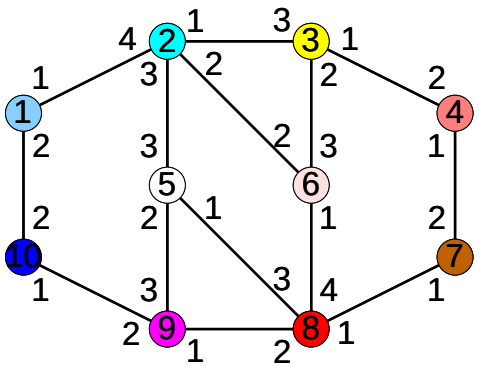
Routing Strategies

Routing Strategy:

- Global preprocessing-algorithm that initializes the data structures.
- Distributed algorithm (“routing scheme”) that runs on every node and performs the routing.



Routing: Models



Addresses:

- Part of the input: Labeled Routing
- Assigned to the nodes by the algorithm: Name Independent Routing

Port Numbers:
Always integers from 1 to $\deg(v)$.

- Part of the input: Fixed Port Model
- Assigned to the edges by the algorithm: Designer Port Model

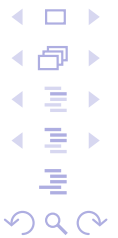
Headers:

- Non-writable/Writable

Performance Measurements

- Local memory size
- Total memory size
- Address/header size
- Stretch (ratio length of routing path / length of shortest path)
- Routing time per node
- Time needed to initialize network

Trade-off: Local memory $\leftrightarrow$ Stretch.
Compact Routing Schemes use $o(n)$ bits of local memory.



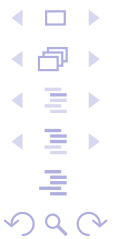
Some Results

Lower bounds:

- Stretch $< 3 \Rightarrow \Omega(n)$ bits of local memory [Gavoille and Gengler, SIROCCO 1997].
- Loop-free with addresses from 1 to $n \Rightarrow$ no scheme with $O(\sqrt{n})$ bits of local memory for every stretch, even in trees [Eilam et al., PODC 1998].

Upper bounds:

- Shortest path routing in trees with $O((\log n)^2 / \log\log n)$ bits local memory [Thorup and Zwick, SPAA 2001].
- Stretch 3 with $O(\sqrt{n}(\log n)^3 / \log\log n)$ bits local memory [Abraham et al., SPAA 2004].



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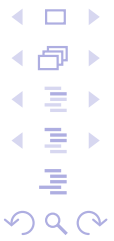
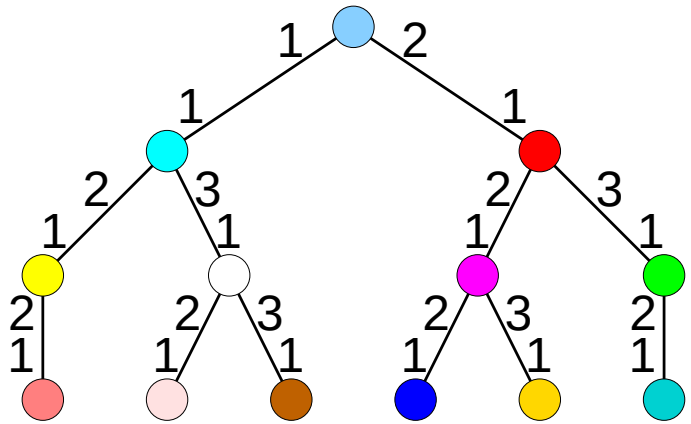
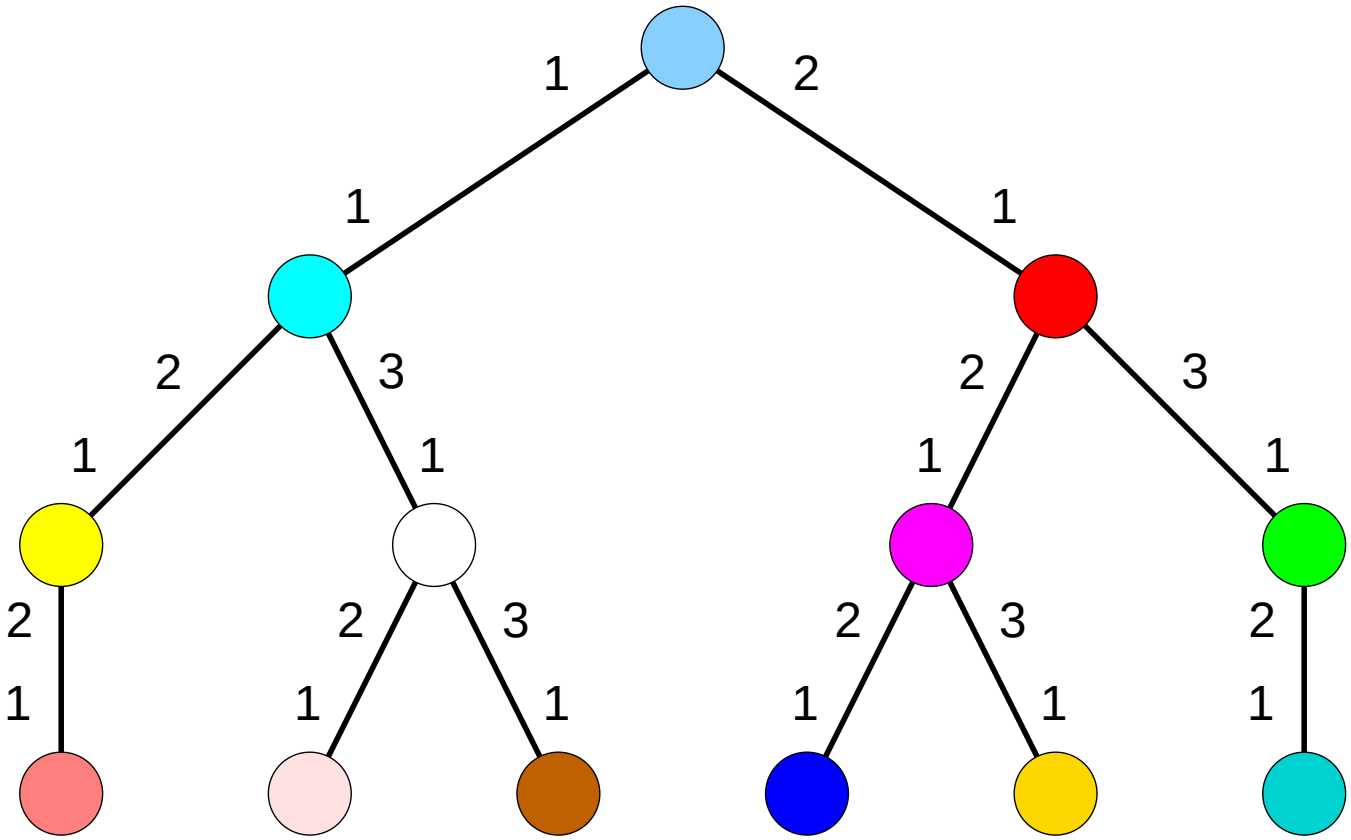


Labeled Routing in Trees



- Interval Routing Scheme by Santoro and Khatib
  [SIAM Journal on Computing, 1985].
- Needs $O(\deg(v) \cdot \log n)$ bits of local memory.

Assigning Addresses to Nodes via DFS



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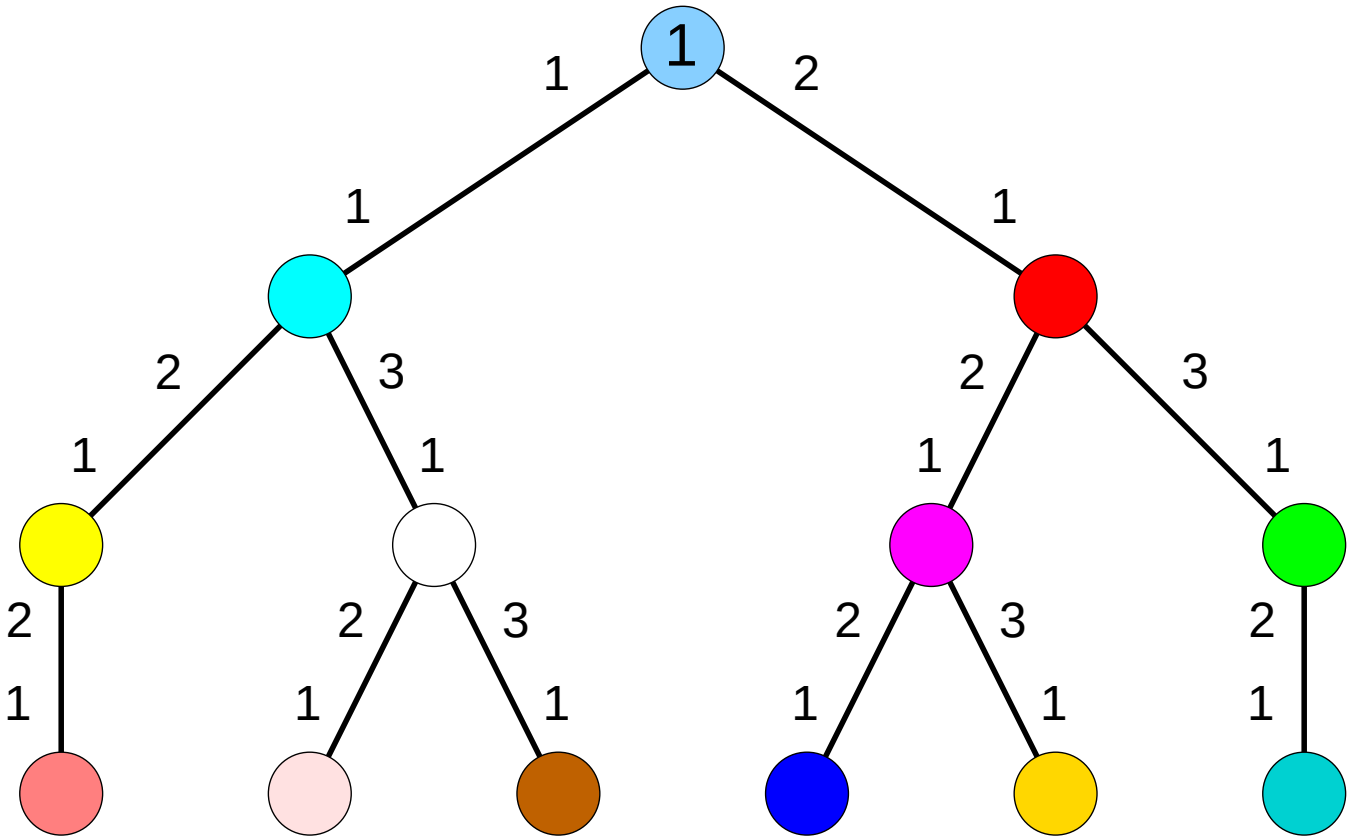

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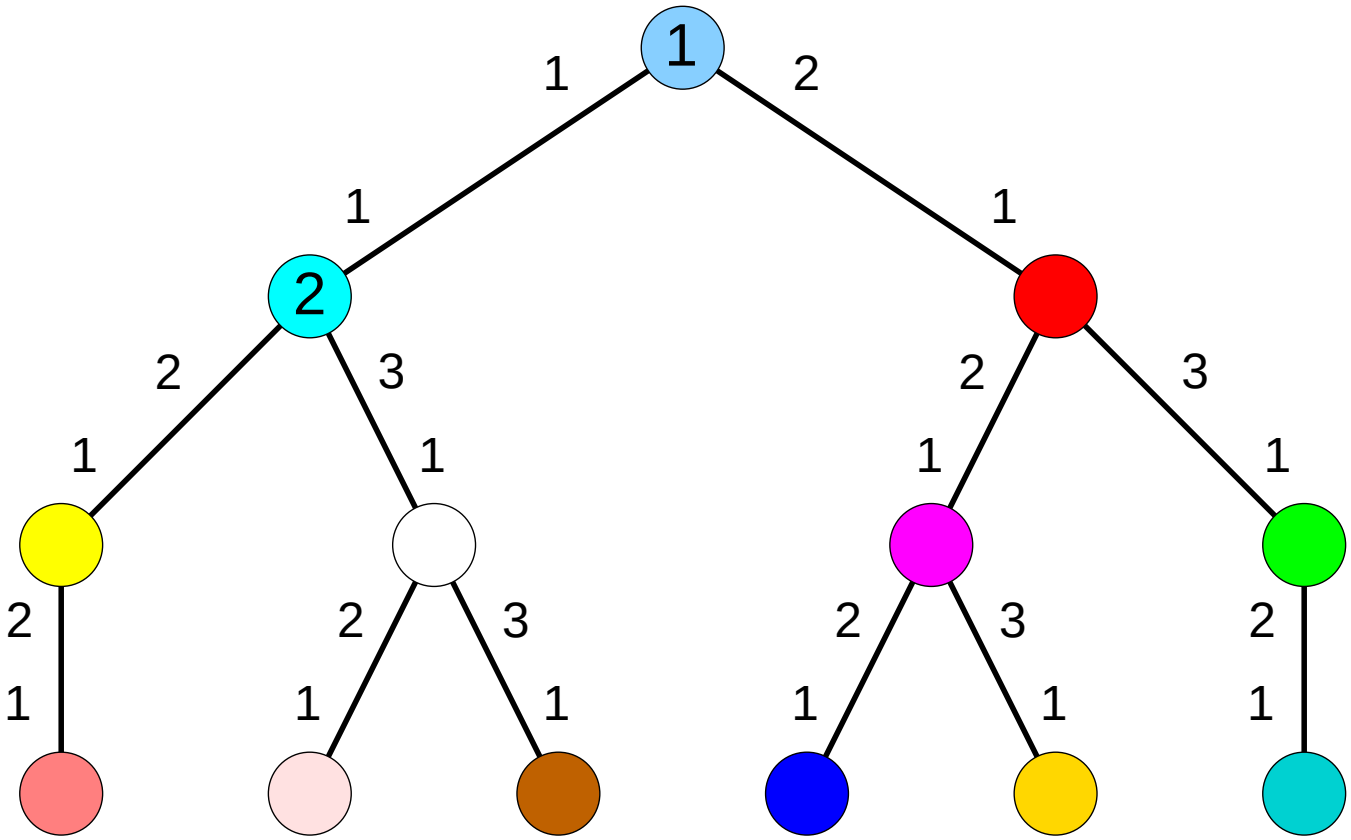

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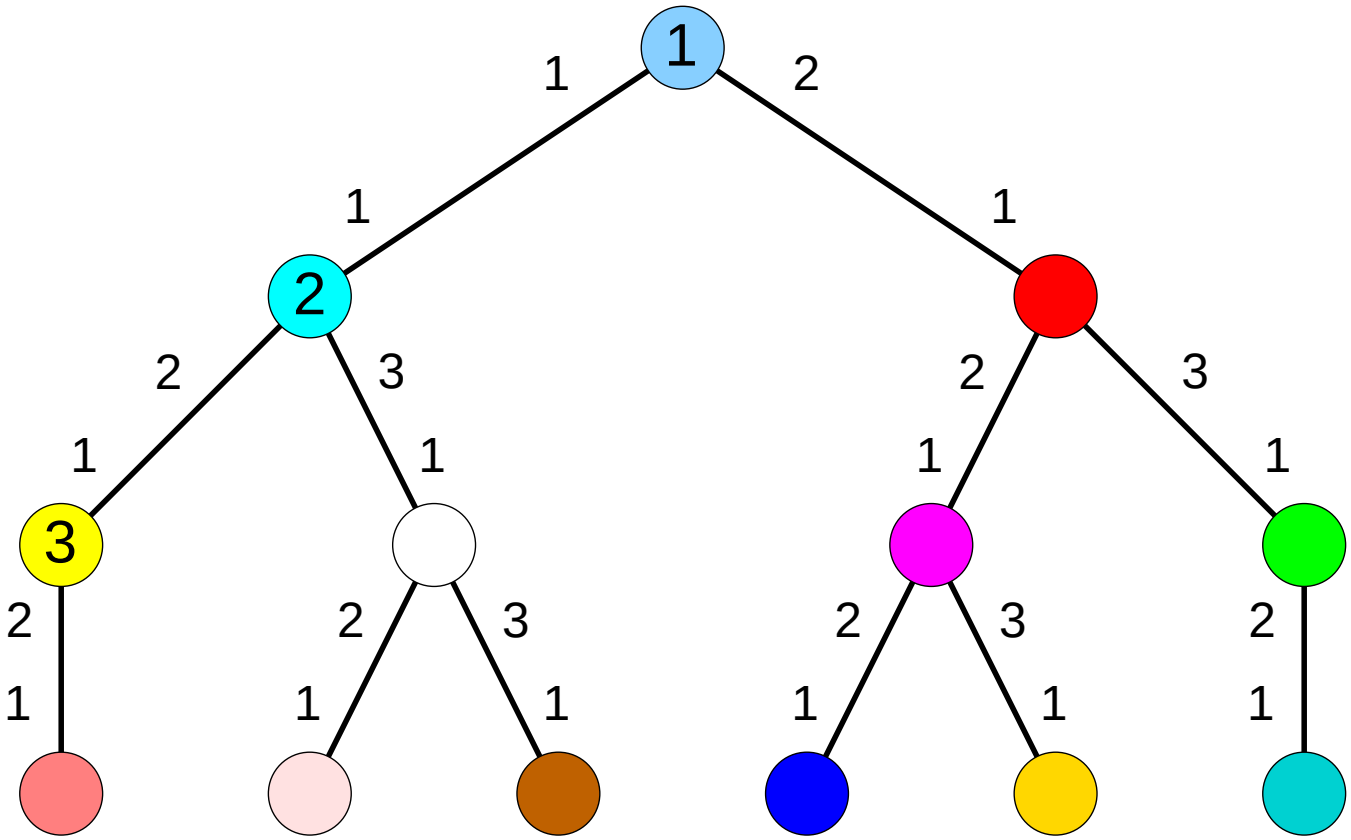

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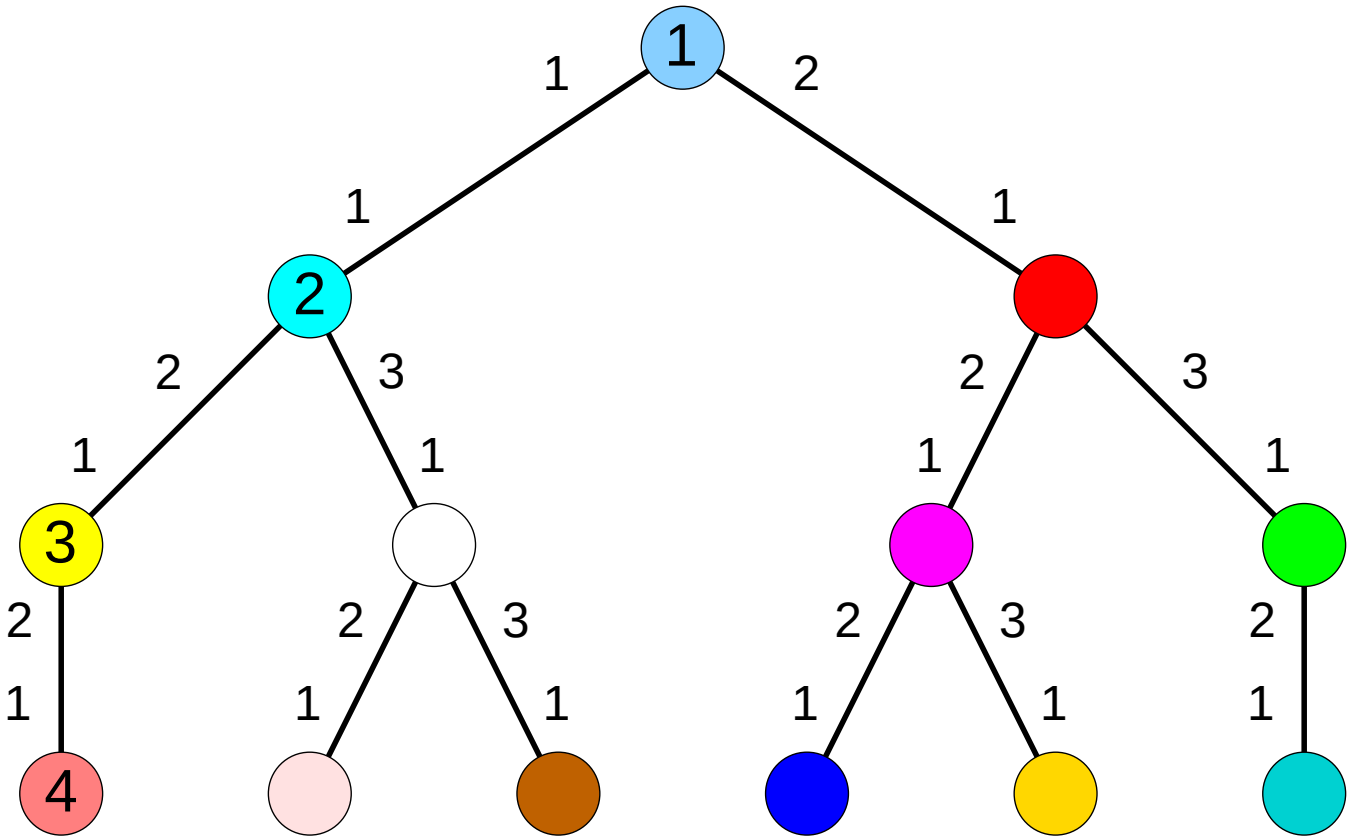

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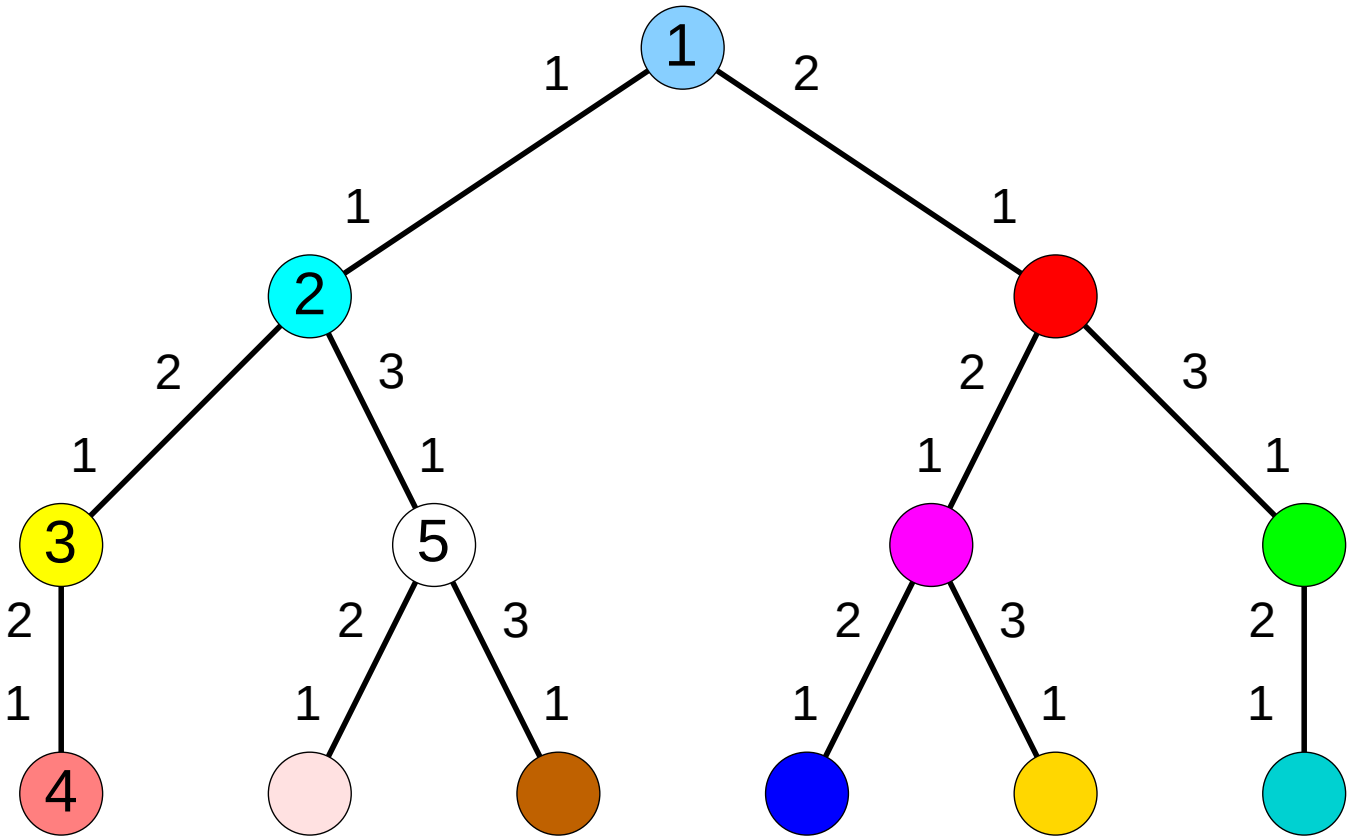

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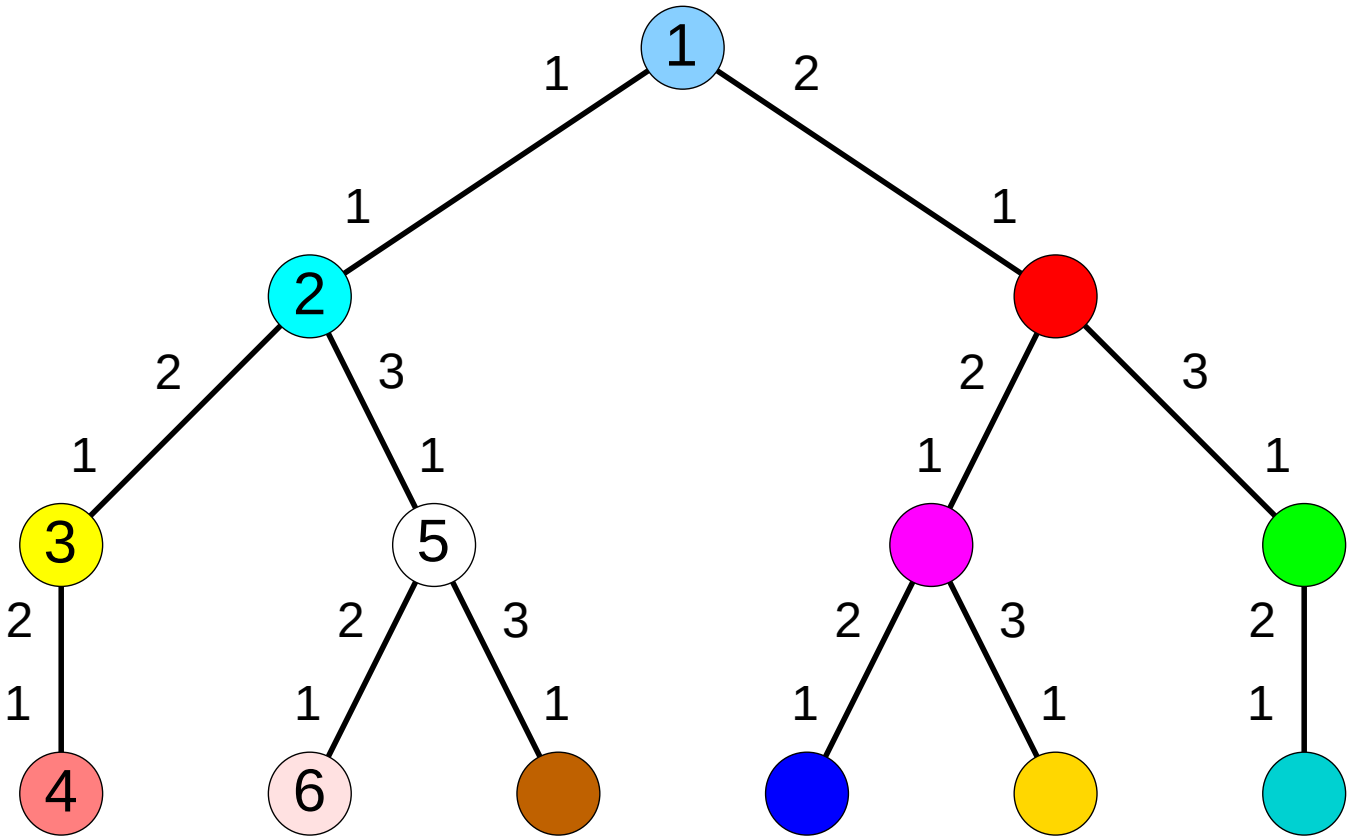

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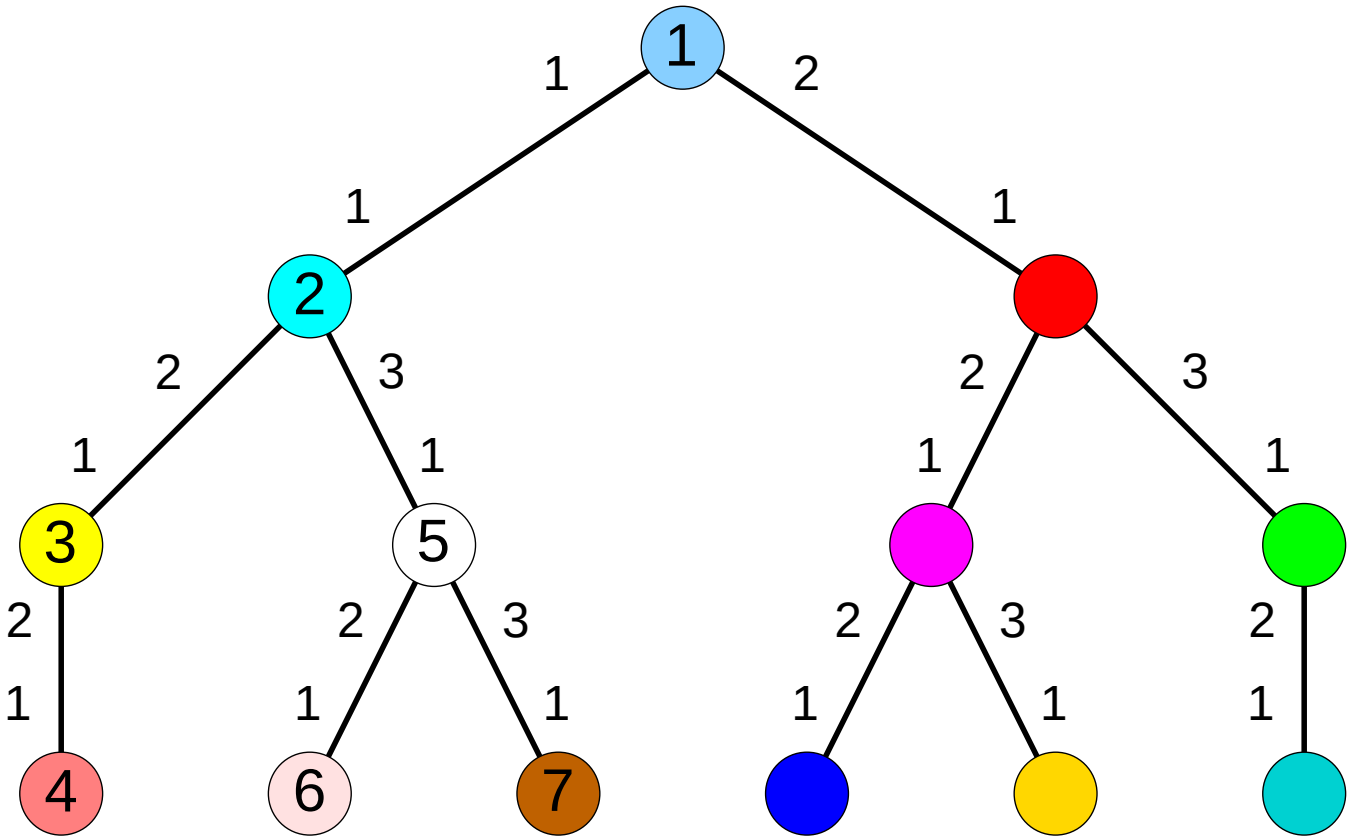

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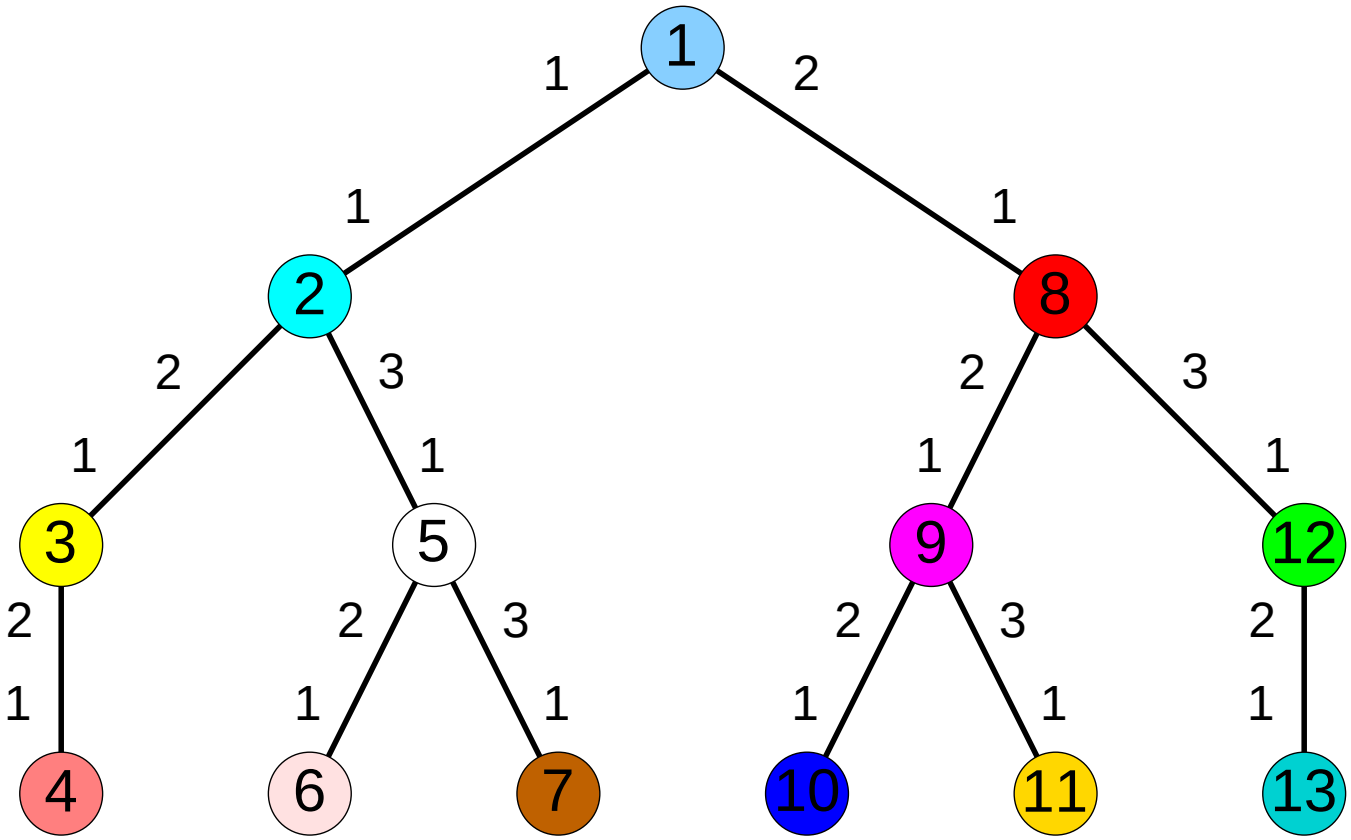
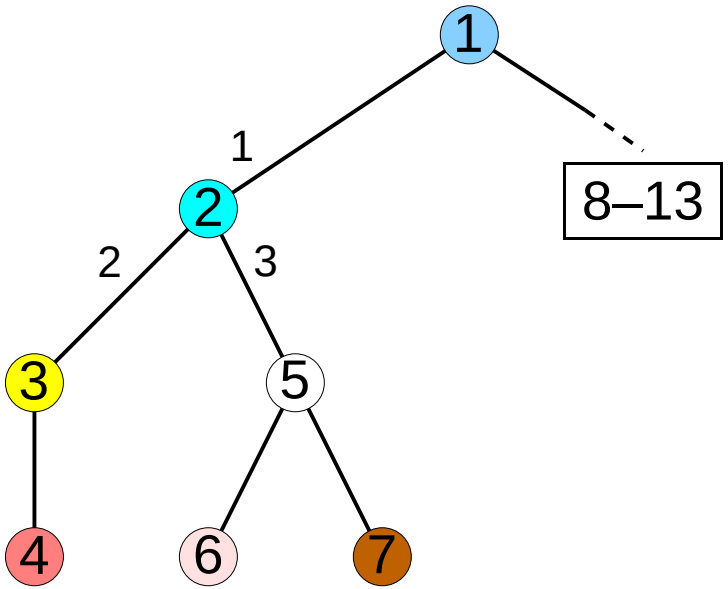
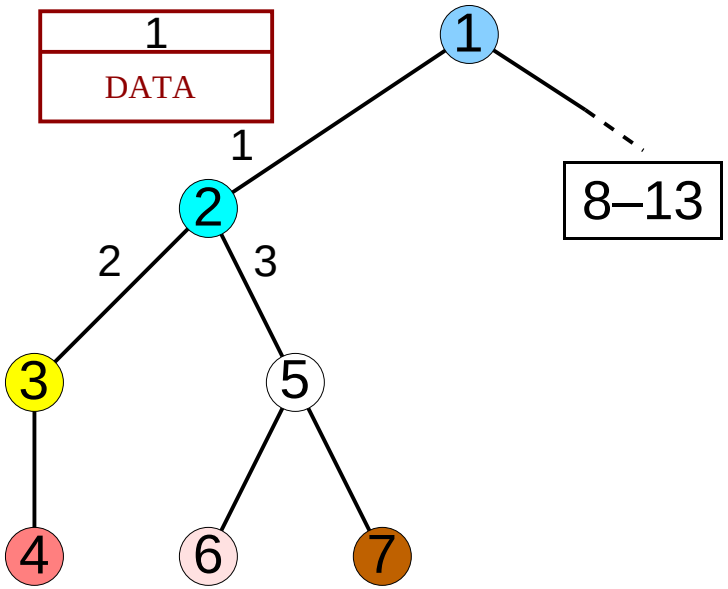


The Local Memory



Local memory of node 2**:**	
Own address:	2
Highest addr. in subtree:	7
Port to parent:	1
Port to subtree:	(4, 2)
Port to subtree:	(7, 3)

Routing Data Packages



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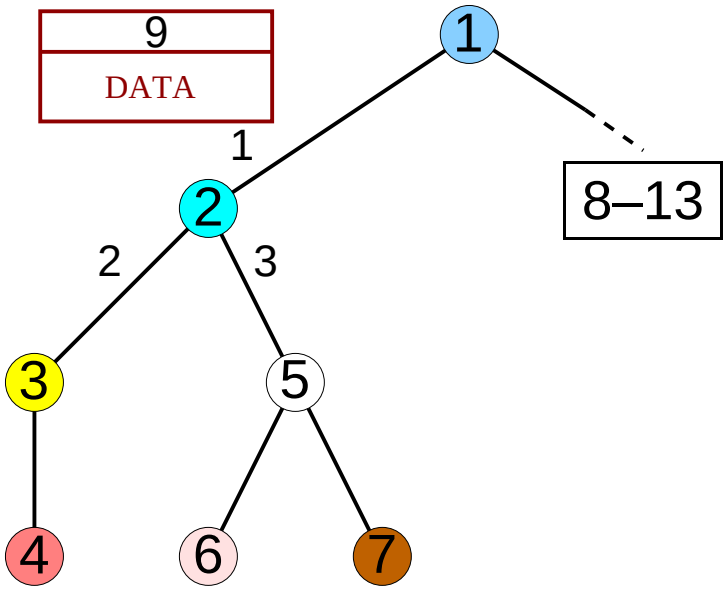

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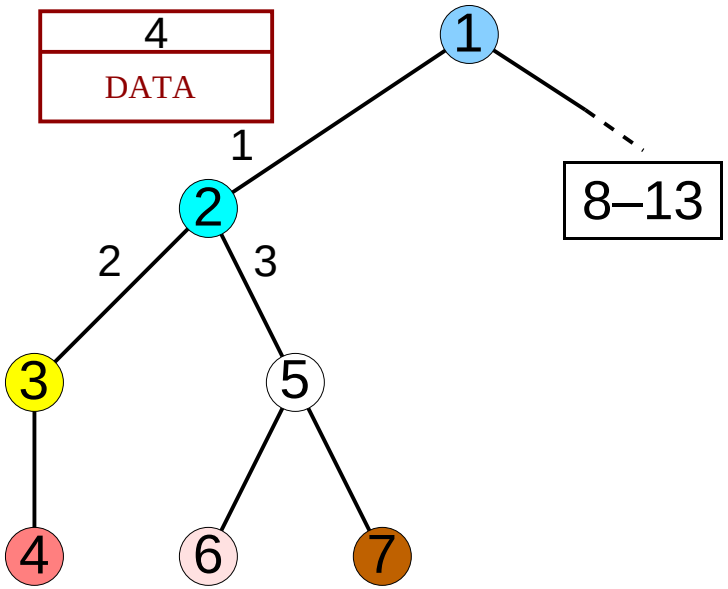

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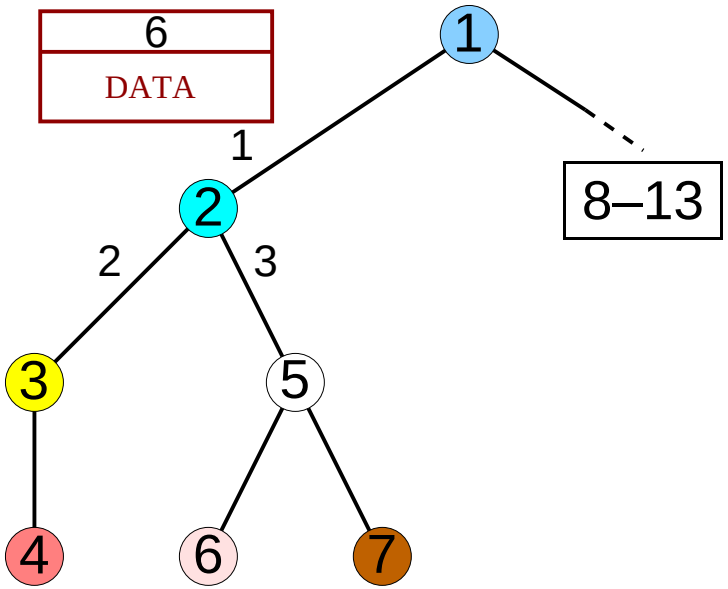

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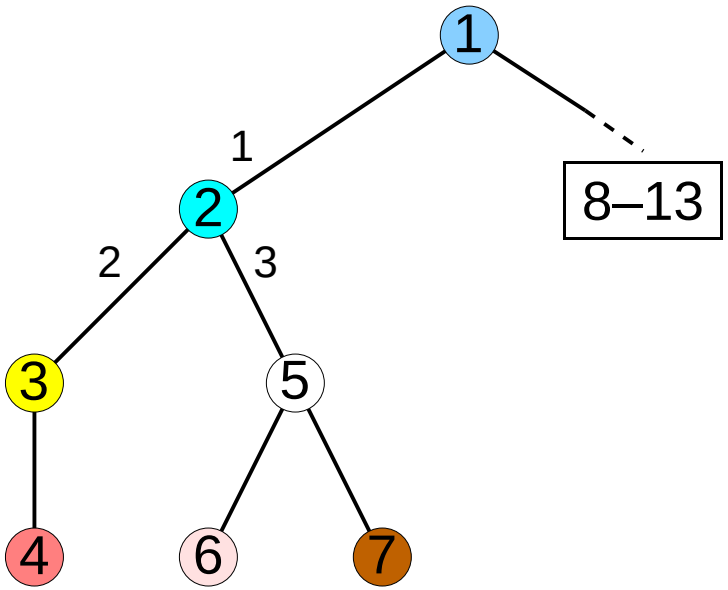
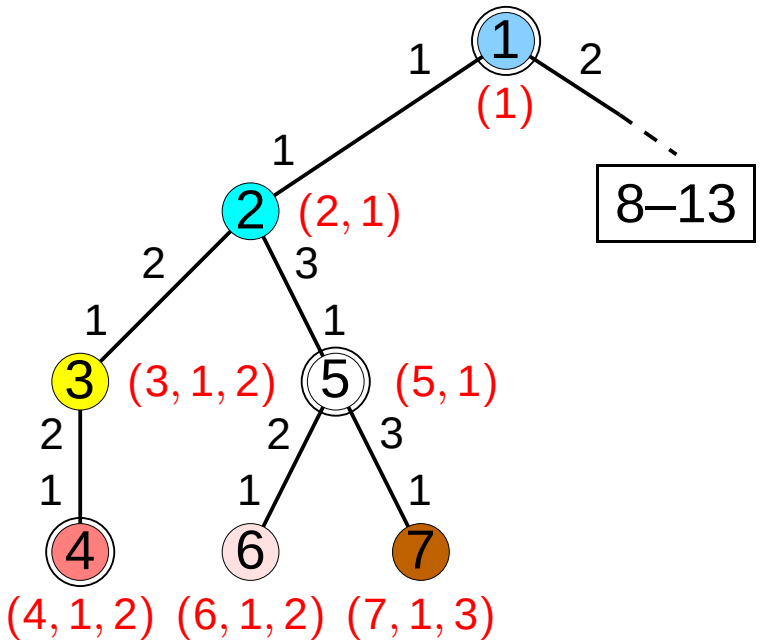
Properties of the Routing Strategy



Local memory of node 2**:**	
Own address:	2
Highest addr. in subtree:	7
Port to parent:	1
Port to subtree:	(4, 2)
Port to subtree:	(7, 3)

- Address size: $\log n$ bits
- Local memory size: $O(\deg(v) \cdot \log n)$ bits
- Can be improved to size of addresses and local memory $O((\log n)^2 / \log\log n)$ bits.

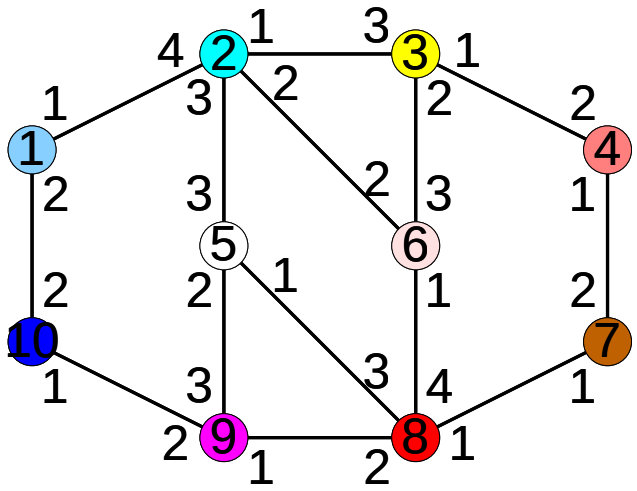
Storing Informations in the Addresses



Local memory of node 2**:**	
Own DFS number:	2
Highest DFS no. in subtree:	7
DFS Number of “heavy” child:	5
#“light” nodes on path from root:	1
Port to parent:	1
Port to heavy child:	3

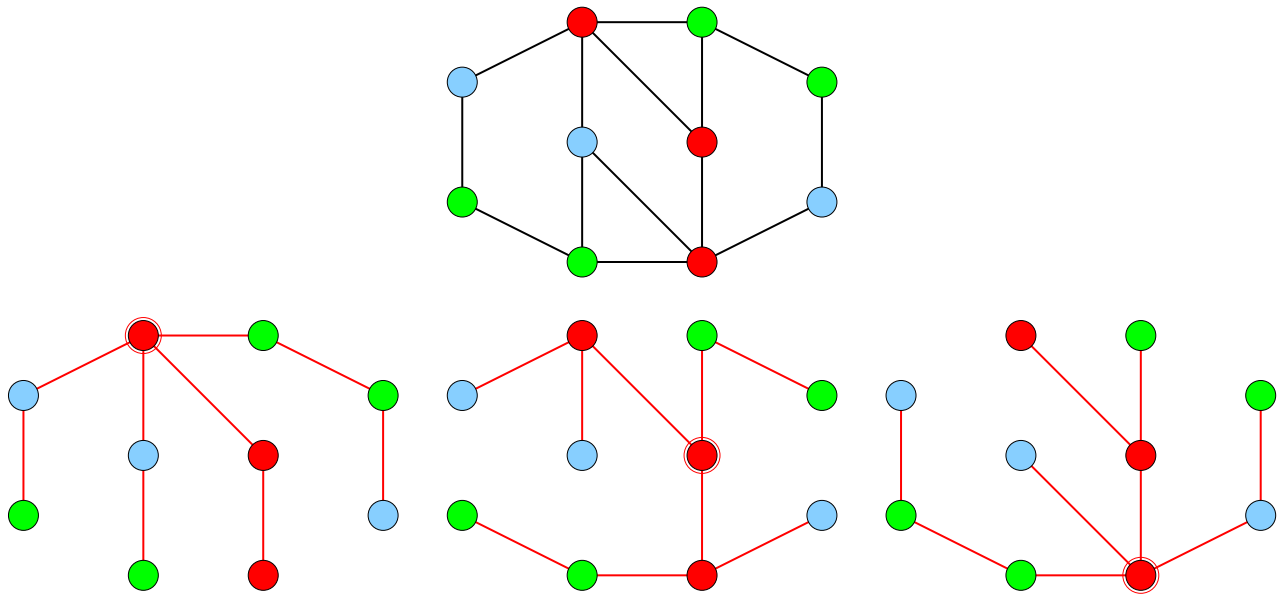
[Thorup and Zwick, SPAA 2001]

A Universal Name Independent Routing Scheme



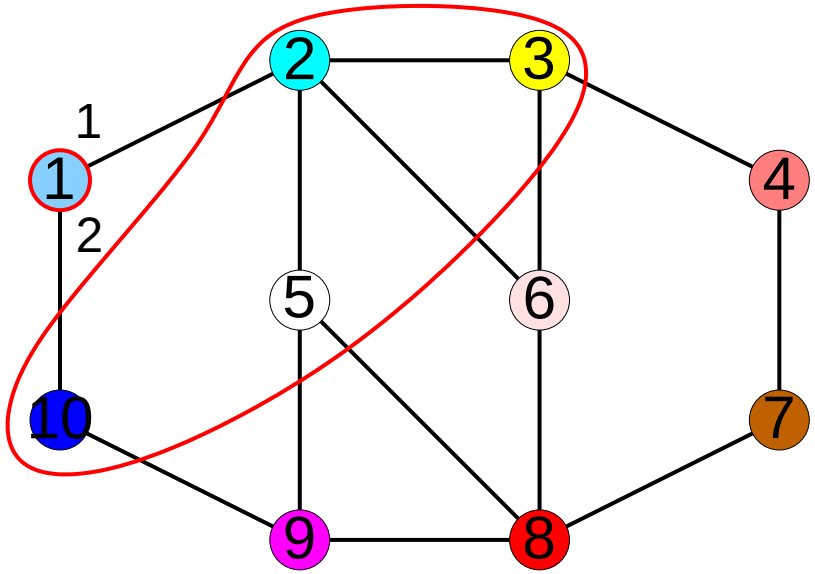
- Scheme of Abraham et al. [SPAA 2004].
- The best known name independent routing scheme with stretch 3.
- Needs $O(\sqrt{n}(\log n)^3 / \log\log n)$ bits of local memory.
- Writable headers of size $O((\log n)^2 / \log\log n)$ bit.

Main idea: Use spanning trees!



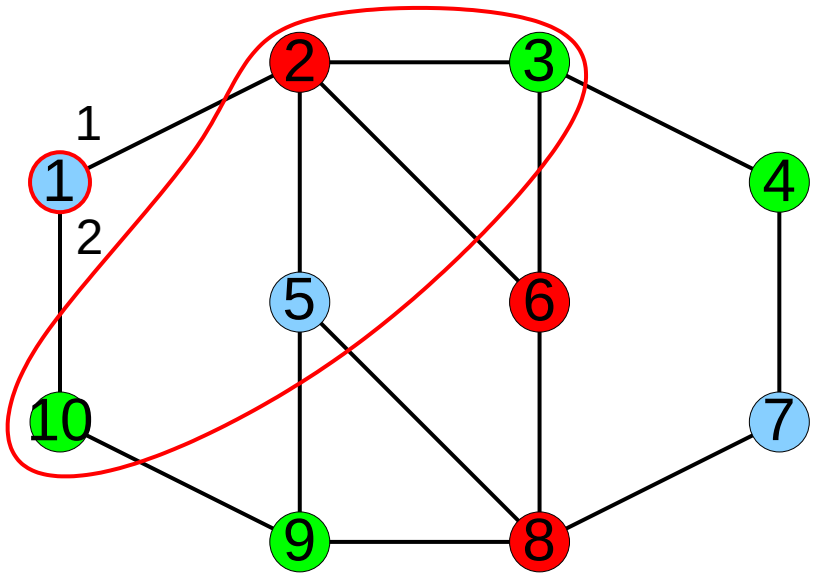
- Several spanning trees.
- For each tree: Routing information of size $O((\log n)^2 / \log \log n)$ bit in local memory.
- Problem: The routing scheme for trees is a labeled routing scheme!

Ingredients



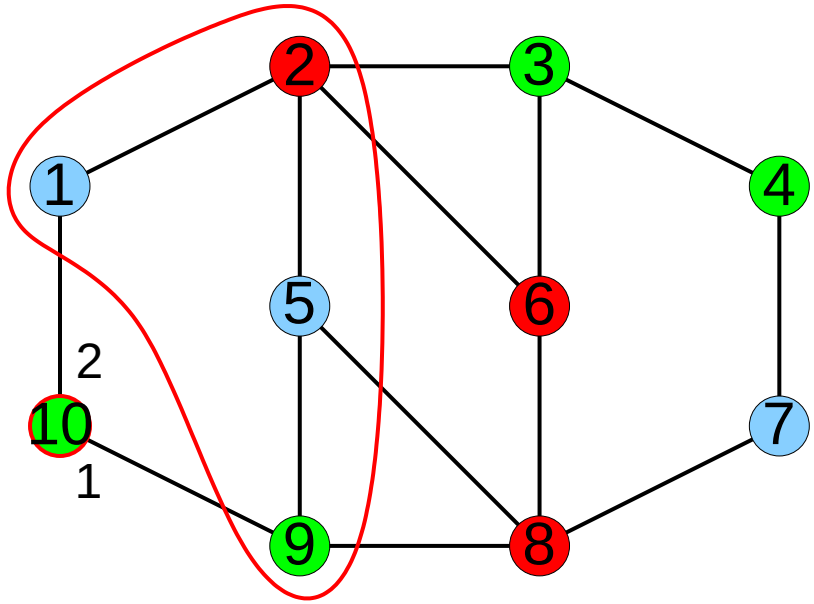
- The *vicinity* $B(v)$ of a node v is the set of the $b\sqrt{n}\log n$ nearest nodes with b being a constant.

Ingredients



- The *vicinity* $B(v)$ of a node v is the set of the $b\sqrt{n}\log n$ nearest nodes with b being a constant.
- There is a coloring with $\sqrt{n}$ colors such that in each $B(v)$ there is at least one node from each color. There are at most $2\sqrt{n}$ nodes of each color.

Ingredients



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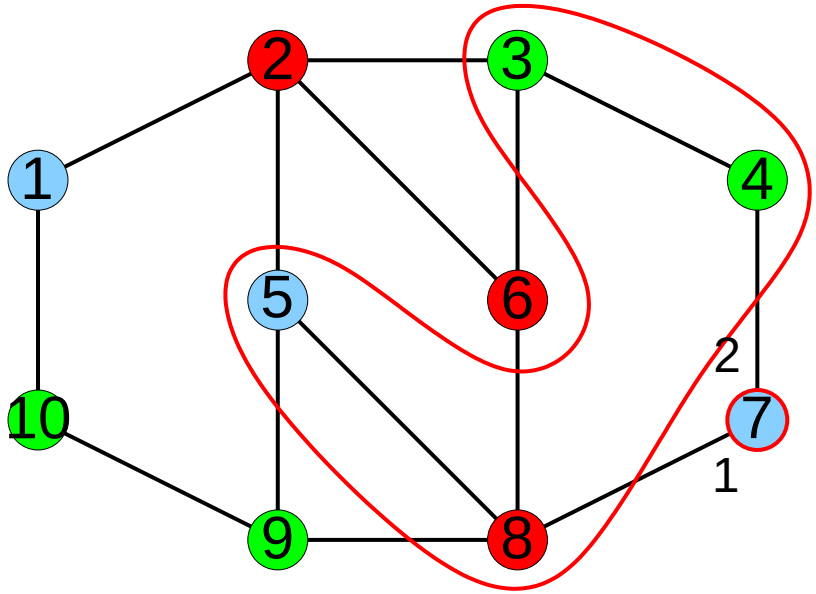

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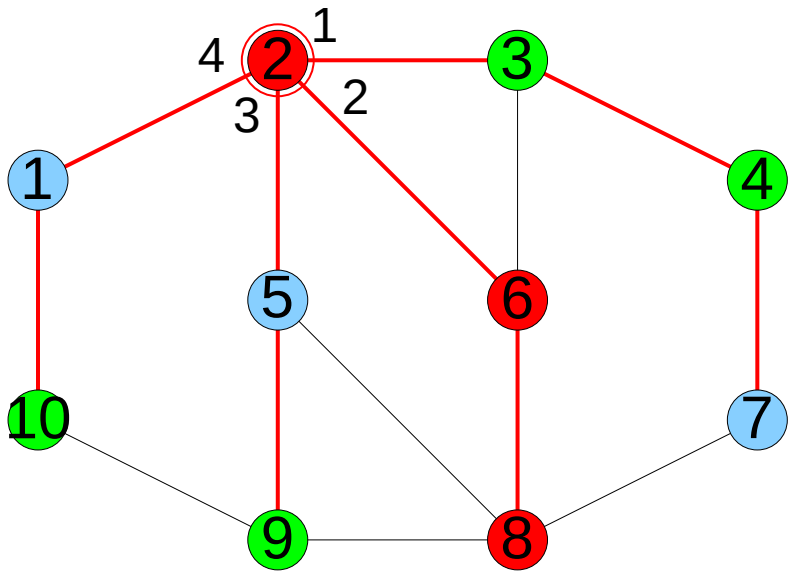
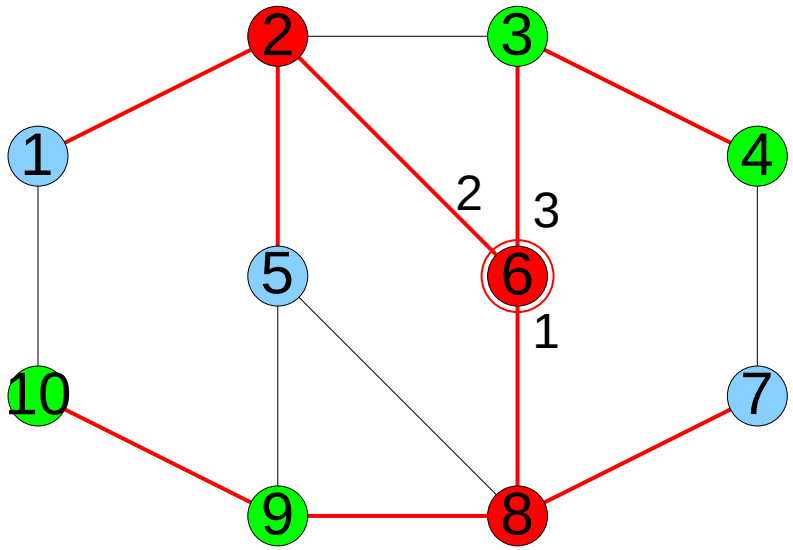


- For each red node i, we denote with T_i the tree consisting of the shortest paths from i to all other nodes.

Ingredients



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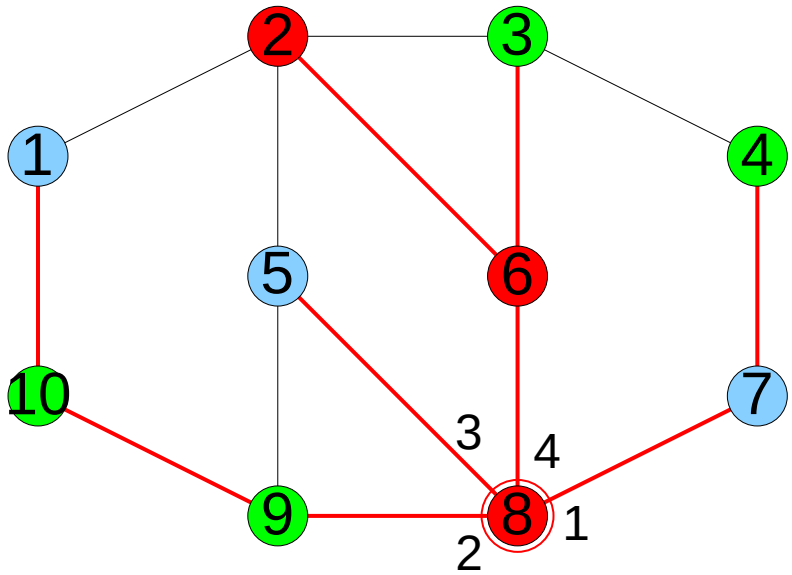

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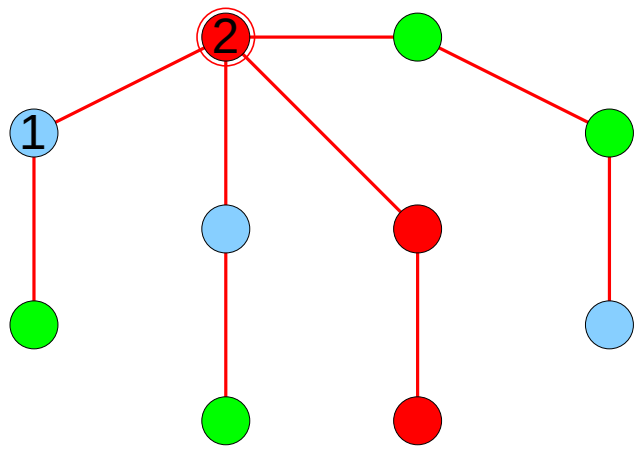
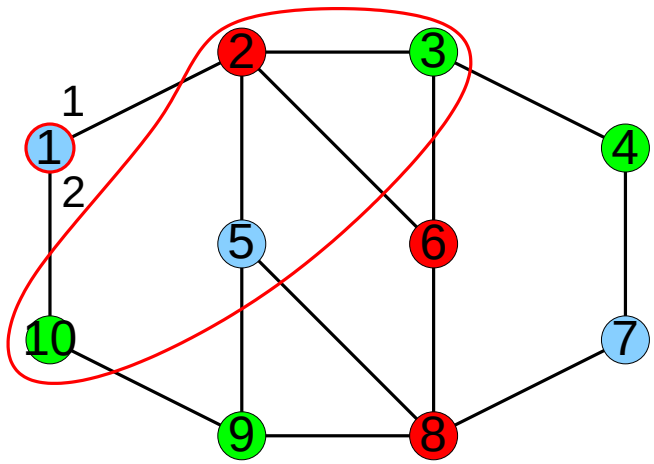
The Local Memory



Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
Port for node 3 $\in B(1)$:	1
Port for node 5 $\in B(1)$:	1
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

The Local Memory



Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
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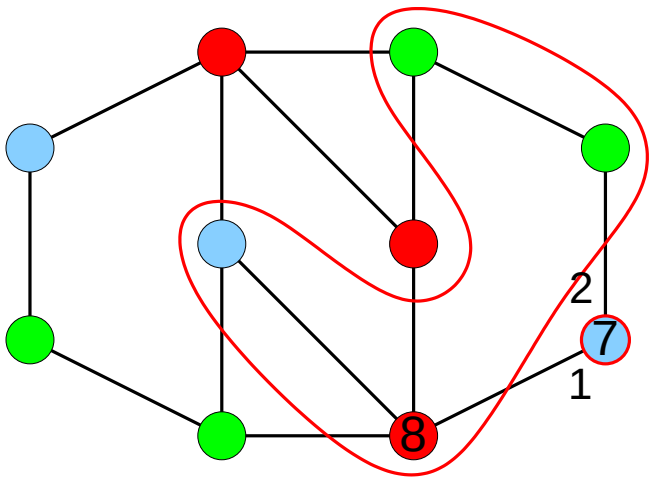

Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
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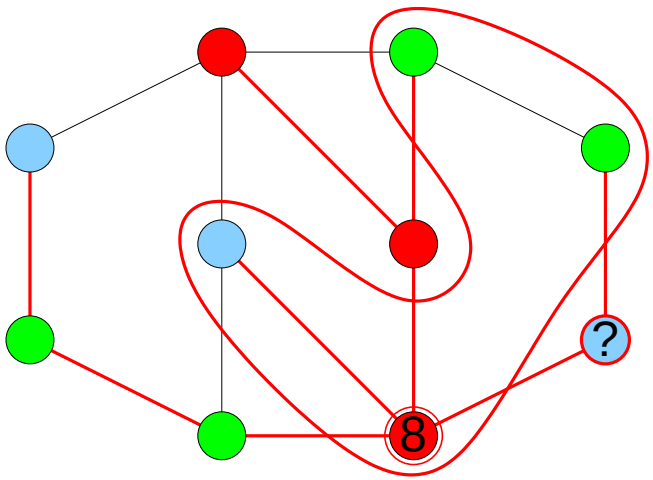

Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
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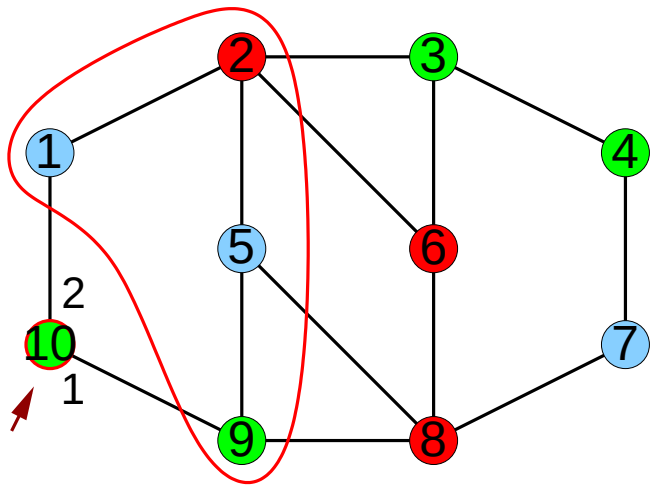
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

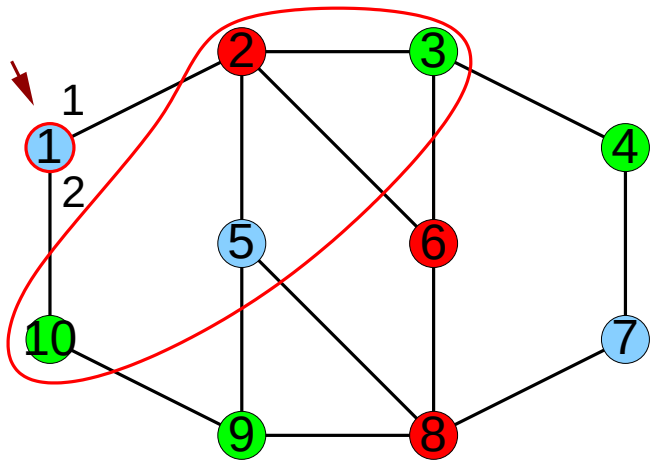
Routing Data Packages



7
DATA

Local memory of node 10:	
Own address:	10
Local memory of node 10 for labeled routing in T_2:	
...	...
...	...
Local memory of node 10 for labeled routing in T_6:	
...	...
...	...
Local memory of node 10 for labeled routing in T_8:	
...	...
...	...
Port for node $1 \in B(10)$:	2
Port for node $2 \in B(10)$:	2
Port for node $5 \in B(10)$:	1
Port for node $9 \in B(10)$:	1
Number of red node in $B(3)$ (same clr.):	2
Address of node 3 (same clr.) in T_2:	...
Number of red node in $B(4)$ (same clr.):	6
Address of node 4 (same clr.) in T_6:	...

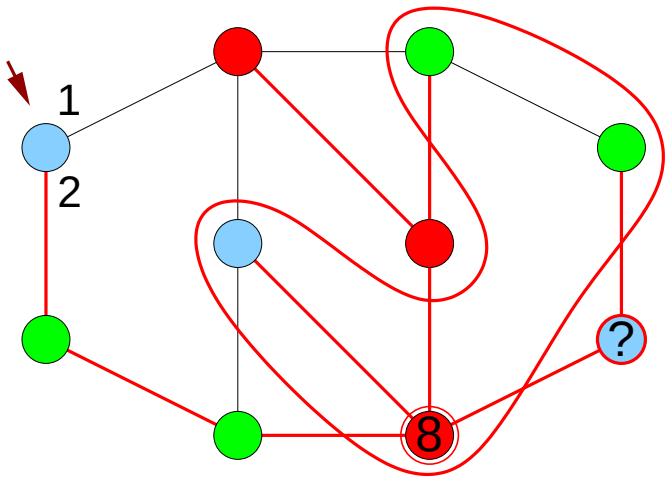
Routing Data Packages



7
DATA

Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
Port for node 3 $\in B(1)$:	1
Port for node 5 $\in B(1)$:	1
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

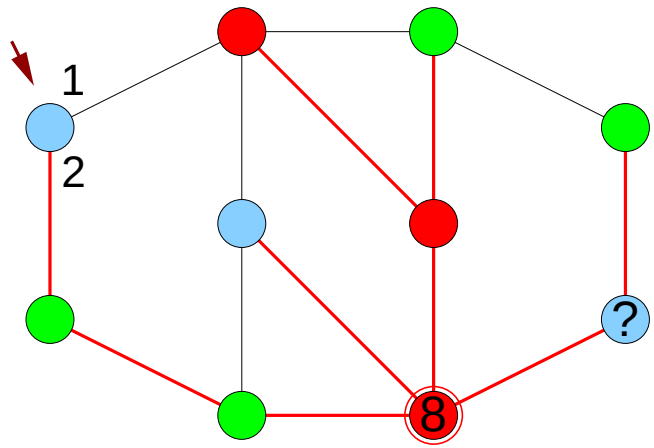
Routing Data Packages



7
DATA

Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
Port for node 3 $\in B(1)$:	1
Port for node 5 $\in B(1)$:	1
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

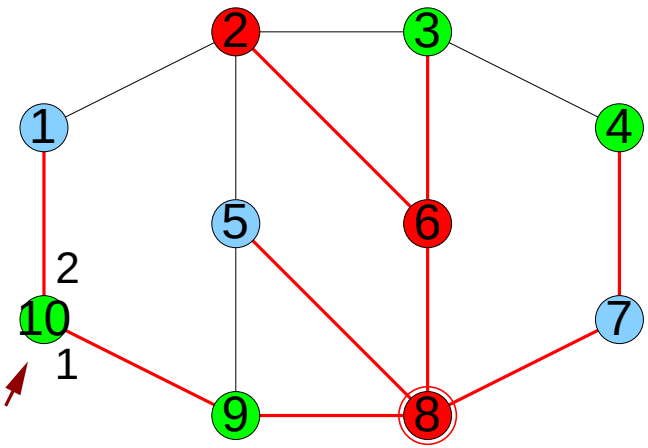
Routing Data Packages



Addr. of 7 in T_8
DATA

Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
Local memory of node 1 for labeled routing in T_6:	
...	...
...	...
Local memory of node 1 for labeled routing in T_8:	
...	...
...	...
Port for node 2 $\in B(1)$:	1
Port for node 3 $\in B(1)$:	1
Port for node 5 $\in B(1)$:	1
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

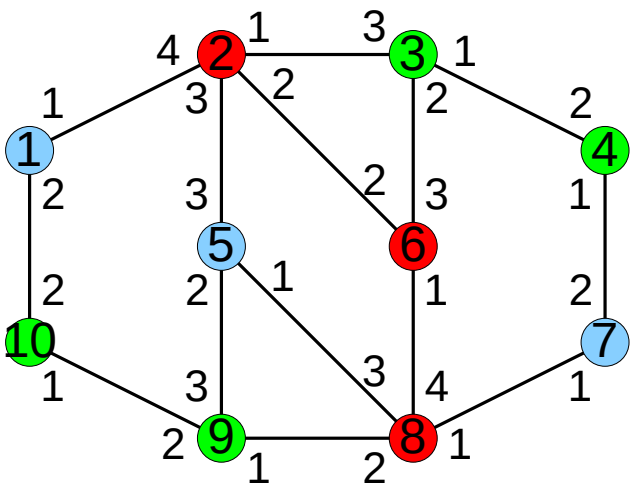
Routing Data Packages



Addr. of 7 in T_8
DATA

Local memory of node 10:	
Own address:	10
Local memory of node 10 for labeled routing in T_2:	
...	...
...	...
Local memory of node 10 for labeled routing in T_6:	
...	...
...	...
Local memory of node 10 for labeled routing in T_8:	
...	...
...	...
Port for node $1 \in B(10)$:	2
Port for node $2 \in B(10)$:	2
Port for node $5 \in B(10)$:	1
Port for node $9 \in B(10)$:	1
Number of red node in $B(3)$ (same clr.):	2
Address of node 3 (same clr.) in T_2:	...
Number of red node in $B(4)$ (same clr.):	6
Address of node 4 (same clr.) in T_6:	...

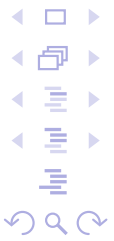
Properties of the Routing Strategy



Local memory of node 1:	
Own address:	1
Local memory of node 1 for labeled routing in T_2:	
...	...
...	...
...	
Port for node 2 $\in B(1)$:	1
Port for node 3 $\in B(1)$:	1
Port for node 5 $\in B(1)$:	1
Port for node 10 $\in B(1)$:	2
Number of red node in $B(7)$ (same clr.):	8
Address of node 7 (same clr.) in T_8:	...

- Header size:
  $O((\log n)^2 / \log\log n$ bits
- Local memory:
  $O(\sqrt{n}(\log n)^3 / \log\log n$ bits
- Stretch 3

Outline



Conclusion

- Low memory consumption even in the case of name-independent routing.
- Applicable to Sensor or Ad-hoc networks?
  - Lower bounds.
  - Is it reasonable to optimize only stretch and local memory consumption?
  - Changing topology: How to update the local data structures?